MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution.

- ☑ They analyze givens, constraints, relationships, and goals.
- ☑ They make conjectures about the form and meaning of the solution
- ☑ They plan a solution pathway rather than simply jumping into a solution attempt.
- ☑ They consider similar problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- ☑ They monitor and evaluate their progress. And change course if necessary.
- ☑ They check their answers to problems using a different method than the one they used to solve the problem.
- ☑ They continually ask, "Does this make sense?"
- ☑ They understand the approaches of others to solving complex problems
- ☑ They identify similarities and differences between different approaches.

Questions that can help students persevere and develop deeper self-awareness of their process, to be asked while they are working

- ☑ Can you explain the situation in your own words?
- ☑ How did people get started on this problem?
- ☑ Have we ever seen a problem like this before? How was the problem similar to this problem?
- ☑ Talk me through what you have done so far, step by step.
- ☑ What is the relationship between the quantities?
- ☑ How will you know if your strategy is working?

Questions that focus student thinking on process, to be asked during class discussions of different student solution methods

- ☑ How did people get started on this problem?
- ☑ How is <student's name> method similar to <student's name> method?
- ☑ How is <student's name> method different from <student's name> method?
- ☑ How do you think <student's name> would use their method to solve this problem?
- ☑ What did you do when you got stuck?
- ☑ How do you know that your answer is correct?
- ☑ Is there another way to solve this problem?
- ☑ Did anyone start with/try a strategy that didn't work?
- ☑ Which one of these strategies helped you see this problem more clearly?
- ☑ What do you appreciate about <student's name>'s strategy?
- ☑ Look over your classmates' work up on the board. What did each student do to make sense of and solve this problem? Can we pull out any general problem-solving strategies that might help us in the future

MP2. Reason abstractly and quantitatively

Mathematically proficient students make sense of the numbers (quantities) and relationships in problem situations

- ☑ They represent abstract situations symbolically – *decontextualize*
- ☑ The manipulate the representing symbols as if they have a life of their own, without attending to their referents
- ☑ They contextualize symbols, pausing to connect them to the situation in the problem
- ☑ They create a coherent representation of the problem

They use the properties of the four operations flexibly

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ What do the numbers in this situation represent? ☑ What does this number represent? (referring to a number appearing a students' work) ☑ Can you make a drawing of the situation? ☑ What does it mean to multiply/divide/add/subtract? ☑ Can you represent the problem with symbols/ equations/ pictures/ sentences/ numbers?

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and established results in constructing arguments.

- ☑ They make conjectures & build a logical progression of statements to explore the truth of their conjectures.
- ☑ They can analyze situations by breaking them into cases, and can recognize and use counterexamples.
- ☑ They justify their conclusions, communicate them to others, and respond to the arguments of others.
- ☑ They can compare the effectiveness of two arguments, and determine correct or flawed logic

Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice

**It is important that students are working on problems that lend themselves to discussion, arguments or critiques*

- ☑ Summarize what <student's name> just said in your own words.
- ☑ How is <student's name>'s answer different from <student's name>'s answer? How are they similar?
- ☑ What questions do you have for <student's name> about their method? (As a follow-up: After a student has presented their work, use the work they put up to ask the rest of the class specific questions…"I see <student's name> did this. What were they thinking here?)
- ☑ What do you appreciate about <student's name>'s method?
- ☑ How can you prove that your answer is correct?
- ☑ Will that always be true?
- ☑ Raise your hand if you agree with Jane. (Count) Now, raise your hand if you agree with Daphne. (Count) Raise your hand if you're not sure. (Count). Ok, so everyone who is unsure is an undecided voter. Everyone else, your job is to convince them to agree with you.
- ☑ Can you come up with some examples that will prove your argument? Or disprove someone else's?
- ☑ Which explanation makes the most sense to you? What did <student's name> do well to make their ideas clear to you?

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems from everyday life, society, and the workplace.

- ☑ They can simplify a complicated situation, realizing that they may need to revise later.
- ☑ They can identify important quantities in a practical situation and show their relationships.
- ☑ They can analyze those relationships mathematically to draw conclusions.
- ☑ They routinely interpret their mathematical results in the context of the situation

They reflect on whether the result makes sense, possibly improving the model if it does not.

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	*It is important that students are working on problems that involve real-world situations ☑ Write a number sentence(s) to describe this situation ☑ How could we draw a picture/make a diagram/visually represent this situation? ☑ What do you already know about solving this problem? ☑ What information would we need to answer this problem? Where could we get that information? ☑ How can you tell if the results make sense? ☑ What factors of the situation did you choose to focus on? Explain your thinking. ☑ What are the practical implications of your findings? Who might be able to use your findings? How might your findings be used by other people?

Example of a Problem Targeting this Mathematical Practice

Wikipedia reports that each day, 8% of all Americans eat at McDonald's. In 2012, there were about 310 million Americans and 12,800 McDonald's restaurants in the United States.
Do you believe the Wikipedia report to be true? Create a mathematical argument to justify your position.

A Word on Mathematical Modelling

> Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.
>
> *A* model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.
>
> (from The Math Assessment Project)

When we hear the word "modelling" in a classroom context, we often think about the teaching strategy where a teacher demonstrates a skill or an approach to a problem for students. When we talk about mathematical modelling, we are talking about something a little different.

To begin to think about mathematical modelling, let's look at two quotes by Henry Pollak

> "When you use mathematics to understand a situation in the real world, and then perhaps use it to take action or even to predict the future, both the real-world situation and the ensuing mathematics are taken seriously."

> "Mathematical modeling begins in the unedited real world, requires problem formulation before problem solving and once the problem is solved, moves back into the real world where the results are considered in their original context. Are the results practical, the answers reasonable, the consequences acceptable? If so, great! If not, take another look at the choices made at the beginning, and try again.
>
> This entire process is what's called mathematical modeling."

(quotes above are from EngageNY PowerPoint on Mathematical Modelling.)

Now, consider the following problem, also from Henry Pollak:

Your grandmother will be arriving at the airport at 6:00 pm. You live 20 miles from the airport. The speed limit is 40 miles per hour. When should you leave to get her?

In a traditional math classroom the answer to this problem would be 5:30, since driving 20 miles at a speed of 40 MPH, will get you to the airport in a half hour.

But if you left your house at 5:30, you would most certainly be late to pick-up your grandmother. What are some other things you might factor in to your calculations?

What about traffic, stop lights, parking, time to meet your grandmother at the baggage claim to help her with her luggage, etc? This begins to get at what we mean by mathematical modelling.

MP5. Use appropriate tools strategically	
Mathematically proficient students consider the available tools when solving a mathematical problem. ☑ They make good decisions about the use of specific tools (calculator, concrete models, digital technology, paper/pencil, ruler, compass, protractor, etc.) ☑ They detect possible errors by strategically using estimation and other mathematical knowledge ☑ They use tools to visualize the results of assumptions, explore consequences and compare predictions with data ☑ They use technological tools to explore and deepen understanding of concepts ☑ They identify relevant external math resources and use them to pose or solve problems	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ Can you draw a picture to show your thinking? ☑ What would be the best tools for working on this problem? (Or offering students a selection of tools and asking them to choose one and then later to explain and reflect on their choice) ☑ What mathematical tool(s) could you use to visualize/represent this situation? ☑ How did it help us to use a _______?

MP6. Attend to precision.	
Mathematically proficient students try to communicate precisely to others. ☑ They try to use clear definitions when discussing their reasoning with others ☑ They express the meaning of the symbols they choose, including using the equal sign consistently and appropriately. ☑ They are careful about specifying units of measure, and labeling quantities in a problem. ☑ They calculate accurately and efficiently. ☑ They express numerical answers with a degree of precision appropriate for the problem context.	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ What does the word ____ mean? ☑ Explain what you did to solve this problem. ☑ How could you label your work to make it clearer? ☑ Is there a more efficient strategy? ☑ How could you organize your work to make it clearer? ☑ How do you know your answer is reasonable? ☑ How exact does your answer need to be? Explain your thinking. ☑ What symbols or mathematical notations are important in this problem? ☑ <Student's name> just explained their strategy to us. What was clear about their strategy? What questions do you have for <Student's name>?

MP7. Look for and make use of structure

Mathematically proficient students look closely for patterns or structure.

- ☑ They recognize quantities can be represented in different ways
- ☑ They can shift back, look at the big picture and shift perspective
- ☑ They can see complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	*Moving from general to specific ☑ How is ____ related to _____? ☑ Is there another way to look at this problem? ☑ What do you know about ____ that would be helpful in this situation? ☑ What patterns do you notice? How can we use that pattern? ☑ How do you know if something is a pattern? ☑ What problems have we done that are similar to this one? How are they similar? ☑ What mathematical concepts/strategies have we learned that helped you work on this problem?

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MP8. Look for and express regularity in repeated reasoning

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<tr><td colspan="2">Mathematically proficient students notice repeated calculations and look for general methods and shortcuts

☑ While working on a problem, mathematically proficient students maintain oversight of the process, while attending to the details.
☑ They continually evaluate the reasonableness of intermediate results
☑ They make generalizations based on findings</td></tr>
<tr><td>Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice</td><td>☑ (Making generalizations) Can you come up with a rule that will help us solve the problem whatever the numbers are?
☑ Will the same strategy work in other situations?
☑ Now that you have the answer, go back and see if there are any patterns you notice.
☑ How can working on this problem help us solve another problem?
☑ Is there another way to solve this problem using less calculation?</td></tr>
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