

## Mathematics Standards Level C

*More than any other, Level C provides the foundation for all future mathematical studies. Fluency with multidigit whole and decimal numbers as well as calculations with fractions (and the relationships between them) carry the most weight at this level. This extends to working with the concept of ratio and rates, addition and subtraction of fractions, and understanding why the procedures for multiplying and dividing fractions make sense. While the greatest emphasis is still on standards for numbers and operations, attention to algebra and geometry increases considerably in Level C. Reading, writing, and interpreting expressions and equations and generating patterns in numbers and shapes provide a conceptual foundation for functions. In addition, analyzing geometric properties, such as parallelism, perpendicularity, and symmetry, and developing and finding volumes of right rectangular prisms take precedence. Level C also emphasizes sampling techniques and data collection through statistical questioning; to previous standards about data, it adds the understanding of measures of center and spread and display of collected data with line plots.*

### LEVEL C (4-5, +6)

#### Number and Operations: Base Ten (+ The Number System)

##### **Generalize place value understanding for multi-digit whole numbers.**

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.* (4.NBT.1)

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (4.NBT.2)

Use place value understanding to round multi-digit whole numbers to any place. (4.NBT.3)

##### **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.5)

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.6)

### **Understand the place value system.**

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $1/10$  of what it represents in the place to its left. (5.NBT.1)

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (5.NBT.2)

Read, write, and compare decimals to thousandths. (5.NBT.3)

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ . (5.NBT.3a)
- Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (5.NBT.3b)

Use place value understanding to round decimals to any place. (5.NBT.4)

### **Perform operations with multi-digit whole numbers and with decimals to hundredths.**

Fluently multiply multi-digit whole numbers using the standard algorithm. (5.NBT.5)

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (5.NBT.6)

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (5.NBT.7) [Note from panel: Applications involving financial literacy should be used.]

## **The Number System**

### **Compute fluently with multi-digit numbers and find common factors and multiples.**

Fluently divide multi-digit numbers using the standard algorithm. (6.NS.2)

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (6.NS.3)

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express  $36 + 8$  as  $4(9 + 2)$ .* (6.NS.4)

**Number and Operations: Fractions<sup>20</sup>****Extend understanding of fraction equivalence and ordering.**

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (4.NF.1)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model. (4.NF.2)

**Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**

Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ . (4.NF.3)

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (4.NF.3a)
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ . (4.NF.3b)
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (4.NF.3c)
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (4.NF.3d)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (4.NF.4)

- Understand a fraction  $a/b$  as a multiple of  $1/b$ . *For example, use a visual fraction model to represent  $5/4$  as the product  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ .* (4.NF.4a)
- Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .)* (4.NF.4b)
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (4.NF.4c)

<sup>20</sup> Expectations at this level in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

**Understand decimal notation for fractions, and compare decimal fractions.**

Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* (4.NF.6)

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model. (4.NF.7)

**Use equivalent fractions as strategy to add and subtract fractions.**

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ . (In general,  $a/b + c/d = (ad + bc)/bd$ .)* (5.NF.1)

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .* (5.NF.2)

**Apply and extend previous understanding of multiplication and division to multiply and divide fractions.**

Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (5.NF.3)

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (5.NF.4)

Interpret multiplication as scaling (resizing), by:

- a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1. (5.NF.5)

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (5.NF.6)

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7)

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for  $(1/3) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(1/3) \div 4 = 1/12$  because  $(1/12) \times 4 = 1/3$ .* (5.NF.7a)
- Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for  $4 \div (1/5)$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (1/5) = 20$  because  $20 \times (1/5) = 4$ .* (5.NF.7b)
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $1/3$ -cup servings are in 2 cups of raisins?* (5.NF.7c)

## The Number System

### Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?* (6.NS.1)

## Ratios and Proportional Relationships

### Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”* (6.RP.1)

Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*<sup>21</sup> (6.RP.2)

<sup>21</sup> Expectations for unit rates at this level are limited to non-complex fractions.

## Operations and Algebraic Thinking

### Use the four operations with whole numbers to solve problems.

Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (4.OA.1)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (4.OA.2)

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (4.OA.3)

### Gain familiarity with factors and multiples.

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (4.OA.4)

### Generate and analyze patterns.

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (4.OA.5)

### Write and interpret numerical expressions.

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (5.OA.1)

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as  $2 \times (8 + 7)$ . Recognize that  $3 \times (2100 + 425)$  is three times as large as the  $2100 + 425$ , without having to calculate the indicated sum or product.* (5.OA.2)

## Expressions and Equations

### Apply and extend previous understandings of arithmetic to algebraic expressions.

Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1)

Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)

- Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract  $y$  from 5” as  $5 - y$ .* (6.EE.2a)
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression  $2(8 + 7)$  as a product of two factors; view  $(8 + 7)$  as both a single entity and a sum of two terms.* (6.EE.2b)
- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ .* (6.EE.2c)

Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .* (6.EE.3)

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.* (6.EE.4)

### **Reason about and solve one-variable equations and inequalities.**

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)

Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers. (6.EE.7)

Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)

### **Represent and analyze quantitative relationships between dependent and independent variables.**

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation  $d = 65t$  to represent the relationship between distance and time.* (6.EE.9)

## Geometry

### **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (4.G.1)

### **Graph points on the coordinate plane to solve real-world and mathematical problems.**

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,  $x$ -axis and  $x$ -coordinate,  $y$ -axis and  $y$ -coordinate). (5.G.1)

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.2)

### **Classify two-dimensional figures into categories based on their properties.**

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (5.G.3)

### **Solve real-world and mathematical problems involving area, surface area, and volume.**

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.3)

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

## Measurement and Data

### **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems



that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.2)

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (4.MD.3)

### **Geometric measurement: understand concepts of angle and measure angles.**

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

- a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $1/360$  of a circle is called a “one-degree angle,” and can be used to measure angles.
- b. An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees. (4.MD.5)

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (4.MD.6)

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (4.MD.7)

### **Convert like measurement units within a given measurement system.**

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (5.MD.1)

### **Represent and interpret data.**

Make a line plot to display a data set of measurements in fractions of a unit ( $1/2$ ,  $1/4$ ,  $1/8$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (5.MD.2) [Note from panel: Plots of numbers other than measurements also should be encouraged.]

### **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units. (5.MD.3)

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (5.MD.4)

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (5.MD.5)

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (5.MD.5a)
- Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. (5.MD.5b)
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. (5.MD.5c)

## Statistics and Probability

### Develop understanding of statistical variability.

Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.* (6.SP.1)

Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2)

Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)

### Summarize and describe distributions.

Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.4)  
[Also see S.ID.1]