Mathematics Standards Level E

Themes introduced and developed in earlier levels continue and deepen in Level E. Having already extended arithmetic calculations from whole numbers to fractions and from fractions to rational and irrational numbers, understanding the real number system comes to the fore. Understanding radical expressions, using and interpreting units in problem solving, and attending to precision are important areas of focus. Prior work with proportional relationships and functions expands from linear expressions, equations, and functions to quadratic, rational, exponential, and polynomial. To bridge the gap between algebra and geometry, rates and relationships are applied to density models. Work also advances in geometry, including using congruence and similarity criteria to prove relationships in geometric figures and determining volumes of cylinders, pyramids, cones, and spheres. Basic skills and knowledge of statistics and probability are applied in a modeling context, in which students interpret and compare data distributions and understand issues of correlation and causation.

Note: Making mathematical models is a Standard for Mathematical Practice (MP.4), and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

LEVEL E (High School)

Number and Quantity: The Real Number System

Extend the properties of exponents to rational exponents.

Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N.RN.2)

Number and Quantity: Quantities

Reason quantitatively and use units to solve problems.

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.* (N.Q.1)

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.* (N.Q.3) [Also see 8.EE.4]

Algebra: Seeing Structure in Expressions

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.* (A.SSE.1)

- Interpret parts of an expression, such as terms, factors, and coefficients.* (A.SSE.1a)

Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. (A.SSE.2) [Also see 7.EE.2]

Write expressions in equivalent forms to solve problems.

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (A.SSE.3) [Also see 7.EE.2]

- Factor a quadratic expression to reveal the zeros of the function it defines.* (A.SSE.3a) [Also see 7.EE.2]

Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1) [Note from panel: Emphasis should be on operations with polynomials.]

Rewrite rational expressions.

Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (A.APR.6)

Algebra: Creating Equations

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.* (A.CED.1) [Also see 7.EE.4, 7.EE.4a, and 7.EE.4b]

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* (A.CED.2)

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.** (A.CED.3)

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.* (A.CED.4) [Also see 7.EE.2]

Algebra: Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1)

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A.REI.2) [Also see 8.EE.2]

Solve equations and inequalities in one equation.

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3) [Also see 7.EE.4, 7.EE.4a, 7.EE.4b, and 8.EE.7]

Solve quadratic equations in one variable. (A.REI.4)

Solve systems of equations.

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A.REI.6) [Also see 8.EE.8b]

Represent and solve equations and inequalities graphically.

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10) [Also see 8.F.5]

Functions: Interpreting Functions

Understand the concept of a function and use function notation.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F.IF.1) [Also see 8.F.1]

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F.IF.2)

Interpret functions that arise in applications in terms of the context.

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *For example, for a quadratic function modeling a projectile in motion, interpret the intercepts and the vertex of the function in the context of the problem.** (F.IF.4) [*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.]*

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble*

*n engines in a factory, then the positive integers would be an appropriate domain for the function.** (F.IF.5)

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* (F.IF.6) [NOTE: See conceptual modeling categories.]

Analyze functions using different representations.

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (F.IF.7) [Also see 8.F.5]

Use properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in an exponential function and then classify it as representing exponential growth or decay.* (F.IF.8b) [Also see 8.EE.1]

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (F.IF.9)

Functions: Building Functions

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.* (F.BF.1) [Also see 8.F.4]

Functions: Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

Distinguish between situations that can be modeled with linear functions and with exponential functions.* (F.LE.1)

- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.* (F.LE.1b)
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.* (F.LE.1c)

Interpret expressions for functions in terms of the situation they model.

Interpret the parameters in a linear or exponential function in terms of a context.* (F.LE.5) [Also see 8.F.4]

Geometry: Congruence

Experiment with transformations in the plane.

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G.CO.1)

Geometry: Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity.

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5) [Also see 8.G.2 and 8.G.4]

Geometry: Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* (G.GMD.3) [Also see 7.G.6]

Geometry: Modeling with Geometry

Apply geometric concepts in modeling situations.

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* (G.MG.2) [Also see 7.RP.3]

Statistics and Probability: Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurable variable.

Represent data with plots on the real number line (dot plots, histograms, and box plots). (S.ID.1) [Also see 6.SP.4 and 8.SP.1]

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S.ID.3) [Also see 7.SP.4]

Summarize, represent, and interpret data on two categorical and quantitative variables.

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5) [Also see 8.SP.4]

Interpret linear models.

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) [Also see 8.SP.3]

Distinguish between correlation and causation. (S.ID.9)