

## Mathematics Standards Level D

*Like preceding levels, Level D also emphasizes number sense and operations, but here the attention is on fluency with all four operations with rational numbers—both negative and positive. The foundation for understanding of irrational numbers is built here, including calculation with square and cube roots and solving simple quadratic equations. Another keen area of concentration is algebra and functions: formulating and reasoning about expressions, equations, and inequalities; solving linear equations and systems of linear equations; grasping the concept of a function; and using functions to describe quantitative relationships. Level D is also where understanding and applying ratios, rates, and proportional reasoning—forming a bridge between rational number operations and algebraic relationships—are developed. Building on the geometric analysis in Level C, the focus turns to analyzing two- and three-dimensional figures using distance, angle, similarity, and congruence, and understanding basic right triangle trigonometry. Having worked with measurement data in previous levels, students at this level develop notions of statistical variability and learn to understand summary statistics and distributions. The concept of probability is introduced and developed at this level.*

### LEVEL D (+6, 7-8)

#### The Number System

##### **Apply and extend previous understandings of numbers to the system of rational numbers.**

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (6.NS.6)

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite. (6.NS.6a)
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (6.NS.6b)
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6c)

Understand ordering and absolute value of rational numbers. (6.NS.7)

- Interpret statements of inequality as statements about the relative position of two numbers on a

number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.* (6.NS.7a)

- Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write  $-3^{\circ}C > -7^{\circ}C$  to express the fact that  $-3^{\circ}C$  is warmer than  $-7^{\circ}C$ .* (6.NS.7b)
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.* (6.NS.7c)
- Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars.* (6.NS.7d)

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)

### **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (7.NS.1)

- Describe situations in which opposite quantities combine to make 0. *For example, if a check is written for the same amount as a deposit, made to the same checking account, the result is a zero increase or decrease in the account balance.* (7.NS.1a)
- Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (7.NS.1b)
- Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (7.NS.1c)
- Apply properties of operations as strategies to add and subtract rational numbers. (7.NS.1d)

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (7.NS.2)

- Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (7.NS.2a)
- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers, then  $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world contexts. (7.NS.2b)

- Apply properties of operations as strategies to multiply and divide rational numbers. (7.NS.2c)
- Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (7.NS.2d)

Solve real-world and mathematical problems involving the four operations with rational numbers. (7.NS.3)

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* (8.NS.2)

**Understand ratio concepts and use ratio reasoning to solve problems.**

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (6.RP.3a)
- Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b)
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (6.RP.3c)
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3d)

**Analyze proportional relationships and use them to solve real-world and mathematical problems.**

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $1/2$  mile in each  $1/4$  hour, compute the unit rate as the complex fraction  $1/2 / 1/4$  miles per hour, equivalently 2 miles per hour.* (7.RP.1)

Recognize and represent proportional relationships between quantities. (7.RP.2)

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (7.RP.2a)
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (7.RP.2b) [Also see 8.EE.5]

- Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .* (7.RP.2c)
- Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate. (7.RP.2d)

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (7.RP.3) [Also see 7.G.1 and G.MG.2]

## Expressions and Equations

### Use properties of operations to generate equivalent expressions.

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (7.EE.1)

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,  $a + 0.05a = 1.05a$  means that “increase by 5%” is the same as “multiply by 1.05.”* (7.EE.2) [Also see A.SSE.2, A.SSE.3, A.SSE.3a, A.CED.4]

### Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* (7.EE.3)

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (7.EE.4) [Also see A.CED.1 and A.REI.3]

- Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* (7.EE.4a) [Also see A.CED.1 and A.REI.3]
- Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.* (7.EE.4b) [Also see A.CED.1 and A.REI.3]

**Work with radicals and integer exponents.**

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,  $3^2 \times 3^{-5} = 3^{(-3)} = (1/3)^3 = 1/27$ .* (8.EE.1) [Also see F.IF.8b]

Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational. (8.EE.2) [Also see A.REI.2]

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.* (8.EE.3)

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4) [Also see N.Q.3]

**Understand the connections between proportional relationships, lines, and linear equations.**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (8.EE.5) [Also see 7.RP.2b]

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

Solve linear equations in one variable. (8.EE.7) [Also see A.REI.3]

- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers). (8.EE.7a)
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (8.EE.7b)

Analyze and solve pairs of simultaneous linear equations. (8.EE.8)

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (8.EE.8a)
- Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6.* (8.EE.8b) [Also see A.REI.6]
- Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (8.EE.8c)

## Functions

### Define, evaluate, and compare functions.

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.<sup>22</sup> (8.F.1) [Also see F.IF.1]

Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* (8.F.3)

### Use functions to model relationships between quantities.

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4) [Also see F.BF.1 and F.LE.5]

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5) [Also see A.REI.10 and F.IF.7]

## Geometry

### Draw, construct, and describe geometrical figures and describe the relationships between them.

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (7.G.1) [Also see 7.RP.3]

### Solve real-life and mathematical problems involving angle, measure, area, surface area, and volume.

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (7.G.4)

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (7.G.5)

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6) [Also see G.GMD.3]

<sup>22</sup> Function notation is not required at this level.

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2) [Also see G.SRT.5]

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4) [Also see G.SRT.5]

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (8.G.5)

**Understand and apply the Pythagorean Theorem.**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

**Statistics and Probability**

**Summarize and describe distributions.**

Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)

**Use random sampling to draw inferences about a population.**

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (7.SP.1)

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling*

*words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (7.SP.2)*

### **Draw informal comparative inferences about two populations.**

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (7.SP.3)*

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book. (7.SP.4) [Also see S.ID.3]*

### **Investigate chance processes and develop, use, and evaluate probability models.**

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around  $\frac{1}{2}$  indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (7.SP.5)

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (7.SP.6)*

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (7.SP.7)

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. (7.SP.7a)*
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (7.SP.7b)*

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (7.SP.8a)

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (7.SP.8b)



**Investigate patterns of association in bivariate data.**

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8.SP.1) [Also see S.ID.1]

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8.SP.2)

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (8.SP.3) [Also see S.ID.7]

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?* (8.SP.4) [Also see S.ID.5]