# The Barbara Ba

By Linda Dacey and Drew Polly

Examine the structure of Common Core State Standards for Mathematics and consider major ideas that will influence their implementation.

## INTRODUCTION TO THE SERIES

This article introduces a series of five articles that *Teaching Children Mathematics* will publish to support mathematics educators as they consider implications of the Common Core State Standards for Mathematics (CCSSM) for instruction and assessment. In this article, the authors examine the structure of the Standards document and its major ideas. Future articles in the series will feature additional topics, ideas, and commentary addressing specific grade bands.

he Common Core State Standards for Mathematics (CCSSM) were "designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers" (CCSSI 2010). For the first time in U.S. history, a set of standards is established that will be taught in nearly every state. Fortyfour states have already adopted CCSSM. Although many teachers are just beginning to learn about this document and its implications for curriculum and pedagogy, as one CCSSM author, Phil Daro, states, "As well designed as these Standards may be, it's just the easy part to design and write something down. The hard part comes ... with putting them to work. And the [teachers] have ultimate control over how they're used."

The CCSSM build on the mathematics Principles and Standards (NCTM 2000) and Curricu-

lum Focal Points (2006) of the National Council of Teachers of Mathematics (NCTM), but they also introduce significant changes to the mathematics we teach and the ways in which we teach it. The CCSSM document emphasizes that these Standards "are not intended to be new names for old ways of doing business" (CCSSI 2010, p. 5).

In this article, we provide a broad overview of the CCSSM by addressing four major parts of the document. First, we describe the Standards for Mathematical Practice, which apply to all students from kindergarten through grade 12. We then describe the parts of the grade-level Standards. Third, we explain how the Standards develop across grade levels. Finally, we discuss the intersection of the domains of the CCSSM.

# What are the Standards for Mathematical Practice?

The eight Standards for Mathematical Practice (see table 1) "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSI 2010, p. 6). These practices describe processes that students are expected to develop and apply in mathematics classrooms. As we consider how the CCSSM will look in classrooms, keep in mind that students in all grades are charged with engaging with mathematics through these eight Standards for Practice. For example, consider the grade 2 Standard, 2.OA.1:

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSSI 2010, p. 19)

Column 1 of **table 1** identifies the eight Standards for Practice and illustrates how they could be integrated while students explore a task. In the following example, we examine how Standard 2.OA.1 (see column 2 of **table 1**) might look in a classroom in light of the Standards for Practice. The Standard asks students to solve addition and subtraction word problems within 100. Let's consider a task:

There are 74 dogs in the dog park. Some more dogs show up. Now there are 131 dogs in the dog park. How many more dogs showed up?

We highly recommend that teachers and teacher-leaders spend time carrying out a similar process with a variety of mathematically rich tasks to consider how these eight Standards for Practice will be implemented consistently in their classroom.

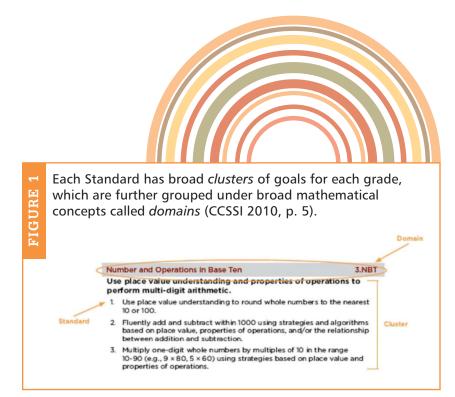
# What are the parts of the CCSSM grade-level Standards?

The CCSSM grade-level Standards have been grouped into clusters, which represent broader mathematical goals. Clusters are grouped into domains, or mathematical concepts. Figure 1 shows the CCSSM diagram identifying various parts of the grade-level Standards. Each set of grade-level Standards includes a two-page introduction describing the critical areas and broad clusters for each grade, supplying teachers with essential ideas to focus on during the year.

Domains represent broad mathematical concepts that are familiar to teachers. Table 2 shows the domains across the grade-level Standards. Some domains in the CCSSM represent a combination of concepts. For example, the Operations and Algebraic Thinking (OA) Domain includes Standards about the four operations, algebraic concepts, and the intersection of those topics. Another combined domain addresses Measurement and Data (MD), which includes Standards in which measurement is used as a context for students to collect, analyze, and interpret data.

# How do the Content Standards develop across grade levels?

Most teachers are interested in gaining an understanding of what their students are to



achieve. Ideally, teachers will work with others at their grade level and have teacher leaders, coaches, principals, or other educators familiar with the CCSSM available for support. Teachers may find it helpful to sort the Standards according to those they are most ready to implement, those they have some ability to implement, and those that need the most preparation to implement. Such a sorting could suggest the basis for an individual or group professional development plan. Particular attention must be given to those areas deemed as critical.

Keep in mind that these Standards were designed to be focused and coherent, delineating the rigor necessary for career and college readiness. Attaining each Standard requires an increasing depth of understanding and proficiency, which takes time to develop. Expectations are explicit and can be tracked across grade levels. For instance, one of the seamless concepts across grade levels in the Operations and Algebraic Thinking (OA) Domain are the

Standard 2.OA.1, which asks students to solve addition and subtraction word problems within 100, might look like this in a classroom in relation to the Standards for Practice.

Practice	How these practices influence 2.OA.2
Make sense and persevere in solving problems	Students make sense of the context and find a way to explore the problem using objects or drawings.
Reason abstractly and quantitatively	Students can decontextualize the problem and match quantities to the numbers of dogs. Further, students can contextualize the quantities and explain what each quantity represents.
Construct viable arguments and critique the reasoning of others	Students clearly and accurately make an argument to defend their strategy. Likewise, students accurately evaluate their classmates' strategies.
Model with mathematics	Students create an equation to match their strategy and work. In our example, this could be $74 + \underline{\hspace{0.5cm}} = 131$ or $131 - 74 = \underline{\hspace{0.5cm}}$ .
Use appropriate tools strategically	Students successfully use objects (e.g., base-ten blocks) or pictures to support their work.
Attend to precision	Students clearly and accurately communicate their process using visual models and written explanations. Their answers are accurate.
Look for and make use of structure	Students apply structure of our base-ten number system to support their work. In this example, students may start at 74 and recognize that if they skip count by tens, they eventually get to 124 and then can count on by ones to reach 131.
Look for and express regularity in repeated reasoning	While solving future problems, students use known information to help them. For example, a student may reason, "I know that when I have a missing addend, I can start with my first number and count up. So I am going to start at 34 and then count up until I get to 131."

varying structures of word problems that are featured in the document's glossary. The structures differ when the location of the unknown number represents the result  $(24+28=\_)$ , the second addend or factor  $(9\times\_=99)$ , or the start number  $(\_+82=141)$ . The explicit recognition of different models of the four operations is grounded in more than twenty years of research on Cognitively Guided Instruction (CGI) (Carpenter, Fennema, and Franke 1996). The OA domain in all grade levels communicates specific strategies for students to use while solving story problems, including pictures, drawings, properties, and equations.

An understanding of individual grade-level Standards will be enhanced by an awareness of how content is developed across the grades. The Standards could support improved curriculum and instruction by increasing focus via the critical areas at each grade level or additional coherence through these carefully developed connections within and across grades. As an example, consider wholenumber arithmetic, which is developed within both the Operations and Algebra (OA) Domain and the Number and Operations in Base Ten (NBT) Domain. Proficiency includes both conceptual understanding and procedural fluency, and the Standards state clear expectations for both. The CCSSM document explicitly calls for students to use drawings, concrete models, place-value strategies, inverse relationships, and properties of operations to solve addition,

# Supporting the CCSSM

Various education leaders in your school system might use these questions to begin conversations that lead to stronger advocacy of the Common Core State Standards for Mathematics.

### **Classroom teachers**

- How might you engage with colleagues to support your understanding of and ability to implement the Content Standards?
- What will the Standards for Practice look like in your classroom?
- How will you establish your classroom culture to embrace these practices?
- How will you ensure learning for all learners?
- Which aspects of your curricula resources can support your work?

### School and district leaders

- What steps have you taken (do you plan to take) to support CCSSM implementation?
- What types of support (e.g., professional development, resources, and so on) will teachers need to successfully implement the CCSSM?
- Who will provide ongoing, in-school support for teachers during implementation?

### **Mathematics educators**

- How will you involve your students in learning experiences that model the Standards for Mathematical Practice?
- How will you support your students' familiarity with research related to these Standards?



Some domains in the CCSSM represent a combination of concepts.

Domains at the grade levels	К	1	2	3	4	5	6
Counting and Cardinality	✓						
Operations and Algebraic Thinking	✓	✓	✓	✓	✓	✓	
Number and Operations in Base Ten	✓	✓	✓	✓	✓	✓	
Measurement and Data	✓	✓	✓	✓	✓	✓	
Geometry	✓	✓	✓	✓	✓	✓	✓
Number and Operations—Fractions				✓	✓	✓	
The Number System							✓
Expressions and Equations							✓
Statistics and Probability							✓
Ratios and Proportional Relationships							✓

# MC<sup>3</sup>: Mathematics Common Core Coalition

In September 2011, eight organizations announced the formation of the Mathematics Common Core Coalition to provide expertise and advice on issues related to effective implementation and assessment of the Common Core State Standards for Mathematics (CCSSM). The members of the coalition are the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of Mathematics Teacher Educators (AMTE), the Association of State Supervisors of Mathematics (ASSM), the Council of Chief State School Officers (CCSSO), the National Governors Association (NGA), the SMARTER Balanced Assessment Consortium (SBAC), and the Partnership for the Assessment of Readiness for College and Careers (PARCC). The coalition will focus on accomplishing the following goals:

- 1. Provide a means to review, research, develop, and communicate common messages throughout the implementation and assessment of CCSSM.
- Provide content and assessment expertise and advice from the communities of mathematics education for the development of the content frameworks of the assessment consortia for CCSSM.
- 3. Collect, analyze, and disseminate information about CCSSM implementation and assessment processes to inform future revisions of the CCSSM.

According to Mike Shaughnessy, president of the National Council of Teachers of Mathematics and chair of the coalition,

The Common Core State Standards present an unusual opportunity to guide and shape the future of mathematics education in the United States. This new Common Core environment also presents real challenges to teachers, districts, and leaders in the education system. The goal of our coalition is to realize the full potential of the Common Core State Standards by combining our strengths and coordinating our efforts to offer the best possible support for teachers and others responsible for delivering high-quality mathematics education to our students.

The Coalition website, www.mathccc.org, will include materials and links to information and resources about CCSSM that the organizations of the coalition are providing to the public and the education community.



subtraction, multiplication, and division tasks before working with customary algorithms. The expectation is that students will develop computational fluency, which CCSSM defines as the ability to compute flexibly, accurately, efficiently, and appropriately. At grades 4–6, these fluency standards also refer to use of a standard algorithm. Table 3 indicates when an arithmetic expectation is introduced and when fluency is expected. Educators should embrace opportunities to examine how content areas develop across the levels.

### How are the domains interconnected?

When thinking about organizing learning experiences for students, understanding connections among different domains is essential. Such connections may indicate Standards that can be considered concurrently or in close proximity. Alternatively, they may suggest opportunities to later reinforce concepts and skills. For example, from grades 1-5, the properties of operationsassociative, commutative, distributive, and identity—are integrated into both the Operations and Algebraic Thinking domain and the Numbers in Base Ten (NBT) domain while working with the four operations. The multiple connections between these two domains help to reinforce the connections between conceptual and procedural knowledge.

Many connections exist between the number-based Standards and the Measurement and Data (MD) domain and the Geometry (G) domain. For example, kindergarten students classify and count objects in the MD domain, which supports counting and cardinality (CC) at that level. Students in grades 2-5 use the operations to solve word problems involving measurement units; and third and fourth graders measure the area of rectangles, reinforcing their understanding of arrays in multiplication. In grades 1–5, line plots and bar graphs are connected to comparison of numbers, number lines, and computation. Partitioning figures into equal shares or areas is included in the geometry domain (G) in grades 1-5, supporting development of the meaning of fractions as parts of regions.

### What are the next steps?

The CCSSM establish common Standards, but—appropriately—do not dictate how they

are to be taught (Daro 2010). Once educators have a common understanding of the expectations, they must consider questions essential for implementation.

As NCTM members who are stakeholders and leaders in mathematics teaching and learning in elementary school classrooms, our role is to ensure that our students have increasing access to rich opportunities to develop and foster their mathematical understanding and practices as we embark into a new chapter of mathematics. This article highlighted the following major points of the CCSSM: the Standards for Mathematical Practice, the organization of the CCSSM Content Standards, the relationship of concepts across grade levels, the evidence-based nature of the Standards, and the intersection of the CCSSM domains. See the sidebar on p. 381 for some questions to keep in mind, depending on your role. It is imperative that we all continue to learn about the implications of these Standards and exert our influence over their implementation.

### **BIBLIOGRAPHY**

Carpenter, Thomas, Elizabeth Fennema, and Megan Franke. 1996. "Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction." The Elementary School Journal (September): 3–20.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for

Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/the-standards.

Daro, Phil. 2011. Common Core Virtual Conference. Video. Upper Saddle River, NJ: Pearson Education. http://commoncore.pearsoned.com/index.cfm?locator=PS16Do.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

. 2006. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence. Reston, VA: NCTM.

National Research Council (NRC). 2001. Adding It Up: Helping Children Learn Mathematics, edited by Jeremy Kilpatrick, Jane Swafford, and Bradford Findell, Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences, and Education. Washington, DC: National Academies Press.

Linda Dacey, Idacey@lesley.edu, is a professor in Education and Mathematics at Lesley University. Her current interests include the creation of tiered mathematical tasks and the investigation of ways to integrate the arts in the teaching of mathematics. Drew Polly, drewpolly@gmail.com, is an assistant professor in the Department of Reading and Elementary Education at the University of North Carolina in Charlotte. His interests include supporting teachers' use of learner-centered mathematical tasks in elementary school classrooms.

BLE 3

Viewing whole-number expectations for arithmetic across grade levels shows that *proficiency* includes both conceptual understanding and procedural fluency.

Level	Concept that is introduced	Fluency that is expected			
K	Add/subtract to 10	Add/subtract to 5			
1	Add/subtract to 100	Add/subtract to 10			
2	Add/subtract to 1000	Add/subtract to 20 (using mental strategies) Add/subtract to 100			
3	Multiply/divide to 100 Multiply 1-digit numbers by multiples of 10 up to 90	Add/subtract to 1000 Multiply/divide to 100			
4	Multiply up to a 4-digit number by a 1-digit number Multiply up to a 2-digit number by a 2-digit number Divide up to a 4-digit number by a 1-digit number	Add/subtract multidigit numbers			
5	Divide up to 4-digit numbers by a 2-digit divisor	Multiply multidigit numbers			
6		Divide multidigit numbers			