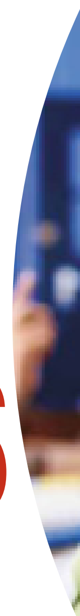
CCSSM: Examining the Critical Areas in Grades 5 and 6



By Drew Polly and Chandra Orrill

Teacher actions and cognitively demanding mathematical tasks for students can simultaneously address grade-level Common Core State Standards for Mathematics as well as the Standards for Mathematical Practice.

To support mathematics educators as they consider implications of the Common Core State Standards for Mathematics (CCSSM) for instruction and assessment, *Teaching Children Mathematics* is publishing a series of feature articles. In this fourth installment, authors Polly and Orrill suggest implementation strategies for grades 5 and 6. A final, cohesive article will appear in the August 2012 issue. Authored by Susan Jo Russell, the last piece concentrates on the implementation of the eight Standards of Mathematical Practice (SMP) and the constellations of Practices and Standards.

Cognitively demanding tasks are at the heart of the implementation of the Common Core State Standards in Mathematics (CCSSI 2010). As with all the grades, teachers of grades 5 and 6 are challenged to use tasks that simultaneously address the grade-level Standards as well as the Standards for Mathematical Practice (SMP). Cognitively demanding tasks require students to examine a mathematical situation, find an entry point to begin their exploration, and apply their understanding of mathematical concepts to find and justify their solutions (Smith and Stein 1998).

In this article, we present two examples that teachers of grades 5 and 6 can use to leverage cognitively demanding tasks with the intention of integrating the grade-level Standards and the SMP. We will also discuss the progression of the Content Standards Domains in grades 5 and 6.

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Cognitively demanding tasks

The authors of the CCSSM document had a vision of students engaging in meaningful mathematics that support student learning of both the practices and content of mathematics. We offer two examples below to illustrate how cognitively demanding mathematical tasks and teacher actions can simultaneously address the CCSSM grade-level Standards as well as the SMP.

Adding fractions

One example of such a task is the Tupelo Township problem (Lappan et al. 2009), in which students are given representations of two one-square-mile sections of land partitioned into different-size parcels, each owned by a different person (see **fig. 1**). The task is designed to (1) help students understand why we use common denominators to add fractions, (2) situate

FIGURE 1

The Tupelo Township problem is from "Bits and Pieces II," a unit in the grade 6 *Connected Mathematics Project 2* series (Lappan et al. 2009).

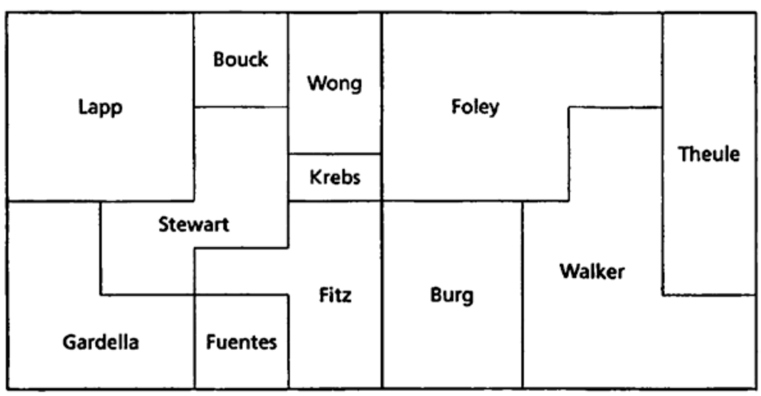


FIGURE 2

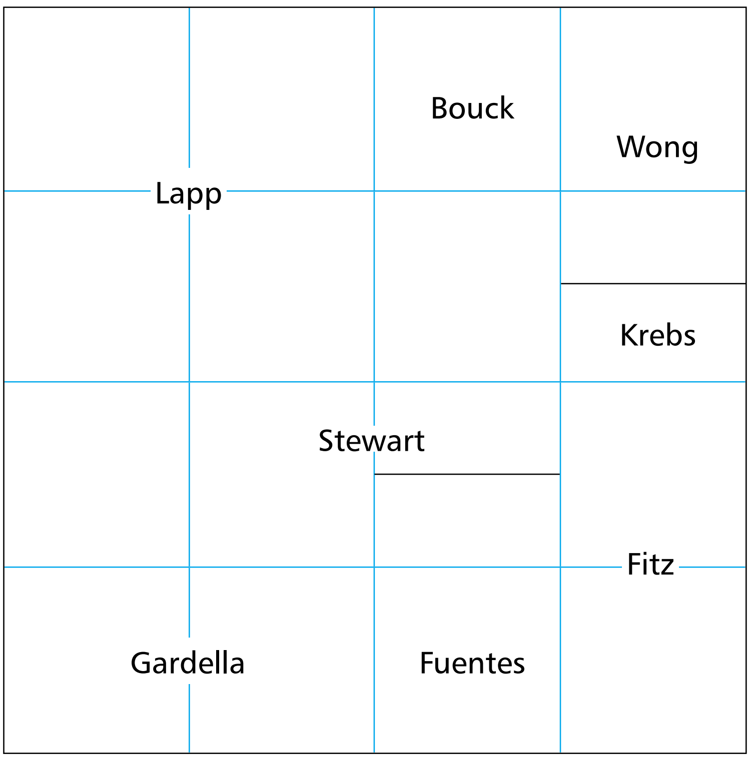
The Tupelo Township problem (Lappan et al. 2009) has students complete various land tasks.

Tulepo Township

1. What fraction of a section does each person own?
2. Find a group of owners whose combined land is equal to 1 1/2 sections of land. Write a number sentence to show your solution.
3. Bouck and Lapp claim that when their land is combined, the total equals Foley's land. Write a number sentence to show whether this is true.
4. Find three people whose combined land equals another person's land. Write a number sentence to show your answer.

FIGURE 3

Students divide Tupelo Township into "Fuentes-size" pieces.



fraction addition in a real-world setting (5.NF.1 and 5.NF.2), and (3) furnish opportunities for students to use fraction multiplication (5.NF.4b and 5.NF.6) or ratio reasoning (6.RP.3) if they are ready. To meet these grade-level Standards, the task asks students to make sense of a problem and persevere in solving it (SMP 1). Situating the task in the context of identifying pieces of land that can be combined in different ways promotes reasoning quantitatively (SMP 2) and constructing arguments (SMP 3). It also ties reasoning about fractions to the representation and requires students to reason about tools that are available and about using them to answer the questions (SMP 5). Students are asked to complete a variety of tasks concerning the land (see **fig. 2**).

To answer the numerous questions that can be asked about the land distribution in Tupelo Township, students must identify the quantity of land each person owns and find ways to combine land (SMP 2). Most students rely on fraction addition for this problem, but some use ratio comparisons to determine the size of individual sections by comparing them to one another.

To start the problem, we offer learners a straightedge. Then we let them decide how they will answer question 1 (see **fig. 2**). Most students quickly see that Lapp owns one-fourth of Section 18, so they write 1/4 in Lapp's section. Some students will see that it takes four Fuentes or Bouck sections to equal one Lapp section. Students can reason about how this relates to the whole by visualizing or drawing in the number of Fuentes-size pieces it will take (see **fig. 3**). Most students intuit that four Fuentes sections fit in a Lapp-size piece and that there are four Lapps, so $4 \times 4 = 16$.

Extending the original boundary lines for the sections highlights for students that the 1/16-size piece is not small enough to determine everyone's land. Students can now discern that the Krebs section is half the size of the Fuentes section and extend the lines to determine Krebs's land ownership. Applying the same procedure, students use the new partitions to define the Stewart and Fitz sections, because the smaller pieces can help determine the irregular areas (see **fig. 4**). However, this partitioning will not work for Section 19 because the widths of Walker's and Foley's land are different from any pieces in Section 18.

Students take a number of approaches to now determine the size of Section 19:

- **Start** from the beginning, looking for lines that can be extended. For example, they may extend the right border of Burg's section upward and the left border of Theule's section downward. This strategy uncovers a piece that looks like a 90-degree rotation of Krebs in Foley's section (see **fig. 5**). Using this rotation, students complete Section 19 with pieces that are 1/32.
- **Divide** both sections into pieces that are 1/64.
- **Use** partitions similar to those in Section 18 where possible and add the rotated pieces where necessary.

Once partitioning identifies the land pieces, students have a much easier time understanding and representing their solution for the first task, finding what fraction of a section each person owns.

Approaches to the second task are interesting because most students begin reasoning about it by thinking of the "unit" as all of Section 18 or all of Section 19. Then they look for half of the other section. In contrast, students less accustomed to reasoning visually about fractions rely on number sentences to find pieces that combine to equal 48/32. Clearly, either approach works. We have seen similar trends for answering the final question. Most students look for pieces that seem to cover the same visual area, and then they use the area values to verify their selections. A few students prefer to rely on traditional fraction addition.

The third task explicitly asks students to construct an argument (SMP 3) about a mathematical situation. Student arguments can be approached from a number of perspectives. For example, they can reason that the Lapp section is as long as Foley's long section and that the Bouck section is as long as Foley's short section. The width of the Walker piece that protrudes into Foley's area plus the width of Theule's section are the same as the width of Stewart's section plus the width of the Krebs section. Therefore, combining Lapp's and Bouck's sections yields the same area as the Foley section.

As with any mathematics tasks, the teacher plays a vital role by asking questions to support students while making connections to important mathematics. The teacher's job is crucial in guiding the debriefing discussion, selecting examples to be used in that discussion, and asking questions that help students make connections among various approaches to the mathematics (Stein et al. 2008). For questions 1 and 2, the teacher might ask students to think about how the strategy to cut the land into small pieces helped. Then she can connect that action to finding common denominators for the fractional parts. Some students used 64ths, and others used 32nds; both strategies present perfect opportunities to examine how common denominators work and why we might choose the least common multiple versus another common multiple for our denominator. For

FIGURE 4

Although Section 18 divides into 32nds, students must modify it to work with Section 19.

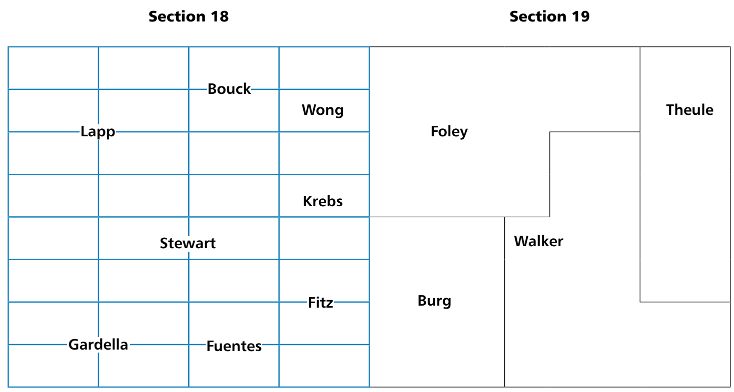
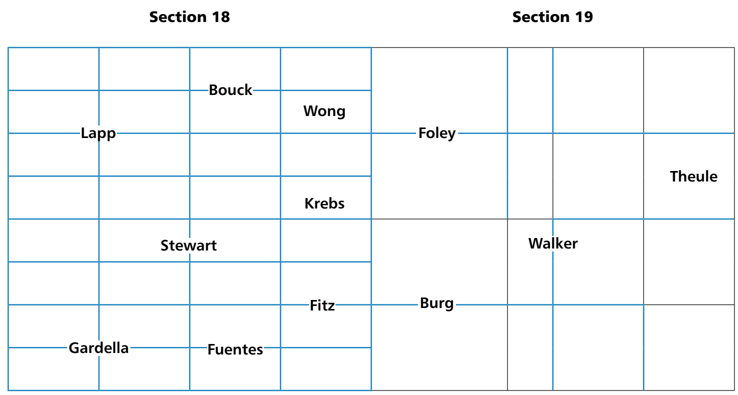


FIGURE 5

Extending the lines in Section 19 and rotating pieces is one strategy to determine its size.

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The Tupelo Township task is most engaging when students must decide the best tools and strategies to use. The Scooping Trail Mix task offers students a chance to defend their strategies. Both problems can support numerous Standards for Mathematical Practice.

question 3, the teacher might ask multiple students to present different responses, and then the class could examine and challenge or support various solution strategies.

This task is most engaging for learners when they are presented with only the diagram, questions, and a straightedge. They must decide how to use the information and tools to find a solution (SMP 5). Students may try a variety of tools available to them while solving, such as overlaying a hundred grid to try to approximate partitions. The task is robust enough that when students realize they are not employing the best tools or strategies, they will quite willingly attempt another approach (SMP 1).

Dividing fractions

Scooping Trail Mix (NCDPI 2012) is a cognitively demanding task (see **fig. 6**) that allows students to explore fraction division (5.NF.7, 6.NS.1). Students divide a whole number by various fractions (grade 5), but the amount of trail mix could be changed to a fraction to address the grade-level Standards for grade 6. The concepts in this task also provide opportunities for students to explore proportions and ratios (6.RP.1, 6.RP.3).

When this task was posed to grade 5 students, they worked in pairs and had access to large bins of trail mix, a measuring cup, plastic sandwich bags, and their mathematics journal to record their work and related mathematical representations. The teacher launched the task by saying, "You may start by scooping trail mix, but the goal is for you to be able to represent your work on paper and explain how dividing fractions works."

Immediately, 24 of the 26 students scooped trail mix to see how many bags they could fill for the various serving amounts. As students began scooping, they commented to one another, "Make sure we fill the entire [measuring] cup," and they assigned roles: One group member scooped, and the other counted bags. The teacher circulated around the room, watching as students used scooping as an entry point to the task. The cognitive demand was maintained by asking, "How can scooping trail mix help you find the answer?" Most students found their solution of a 1/3-cup serving size by scooping trail mix into every bag.

Two students tried a different strategy, using their math journals to draw the problem. Working together, they each drew small boxes representing the fraction 1/3. As they worked, the students used circles to group three small boxes representing 1 cup (see **fig. 7**). They continued this process until they had an equivalence of 5 cups. During the 20 minutes allotted for the task, these two fifth graders used a similar strategy to find the answers for each serving size (1/3, 1/4, and 1/8).

Bringing the class together to discuss the work, the teacher opened the dialogue by saying, "As you share, we are interested in two things: what strategies you used and how you know that your answer makes sense." On the basis of solution strategies she had observed, the teacher had previously identified a few students

FIGURE 6

The Scooping Trail Mix problem develops an understanding about the division of fractions.

Scooping Trail Mix

We have 5 cups of trail mix. While preparing snacks for the field trip, we must put the same amount of trail mix in each bag.

- How many bags can we fill if we put the following amounts in each bag?
  1/3 cup
  1/4 cup
  1/8 cup
- Support each answer with both a picture and an equation.
- Which serving size is the best choice for our class?

whom she wanted to share their work.

Katie's strategy was to scoop 1/3-cup servings into 15 bags. Veronica's approach was to scoop 1/4-cup servings:

Katie: We found that we can fill 15 bags if we put 1/3 of a cup in each bag.
Teacher: How do you know that answer makes sense?
Katie: After we scooped it out, we figured that if we put 1/3 of a cup in a bag, then we could put 1 cup of trail mix in 3 bags. So for every 1 cup of trail mix, we can fill 3 bags. Since we have 5 cups of trail mix, the number of bags is 3 × 5, which is 15.
Veronica: We can fill 20 bags if we put 1/4 of a cup in each bag. I kept scooping 1/4 of a cup out and kept adding up the amount that I scooped. After 20 scoops, we had scooped 5 cups of trail mix.

The teacher wanted to promote mathematical reasoning about proportional relationships, so she guided the class conversation:

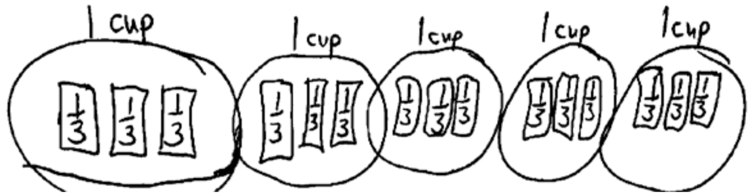
Teacher: Let's go back to what Katie said. Katie told us that if we are scooping 1/3 of a cup in each bag, then 1 cup of trail mix could fill 3 bags. How many bags could we fill if we put 1/8 of a cup in each bag?
Marisia: I scooped 1/8 of a cup 8 times, and then I realized that 1 cup of trail mix will fill 8 bags. That makes sense since 1/8 × 8 equals 1. Then we would multiply 8 × 5 since we have 5 cups of trail mix, and that equals 40 bags.

The teacher expanded on the similarities between Katie and Marisia's explanations:

Teacher: Both Katie and Marisia have talked about finding how many bags we can fill with 1 cup and then multiplying by 5. Why do we need to multiply by 5?
Tyrone: It's like we are trying to find out how many bags can be filled with 1 cup and multiplying by 5 since we actually have 5 cups to share instead of just 1 cup.
Teacher: If we had 10 cups of trail mix instead of 5 cups, how would our approach change?
Tyrone: We would still find out how many bags we can fill with 1 cup but then multiply by 10 instead of by 5.

FIGURE 7

This student used a 1/3-cup serving size.



Teacher: What do the rest of you think about that?
[*Many students murmuring*] I agree. Yes.
Teacher: Did anyone else notice any other relationships they want to share?
Steven: It makes sense that we can fill 20 bags with 1/4 of a cup of trail mix and 40 bags with 1/8 of a cup of trail mix. There are two 1/8s of a cup in 1/4 of a cup. That means we should be able to fill twice as many bags using 1/8 of a cup compared to using 1/4 of a cup. Since 40 is twice as much as 20, that makes sense.
Michael: I'm confused.
Steven: Do you agree [*immediately grabbing the 1/8-cup scoops and the 1/4-cup scoop and scooping 1/8 cup of trail mix and putting it in the 1/4-cup scoop*] that two 1/8 of a cup equal 1/4 of a cup? [*He demonstrates again that two 1/8 cups equal 1/4 cup*].
[*All the students agree with Steven's reasoning.*]
Steven: Since we can fill 20 bags with 1/4 of a cup of trail mix, we just need to double 20 to find out how many bags we can fill with 1/8 of a cup.

To have a meaningful opportunity to construct viable arguments (SMP 3), students received a writing assignment and worked individually in their math journals to create an argument about which serving size was the best choice. When students shared their arguments as a conclusion to the lesson, they practiced using feedback and critique, another component of SMP 3.

The Scooping Trail Mix task can support numerous SMPs. Student work throughout the entire task supports sense making, perseverance, and use of mathematical tools while solving problems (SMP 1, 5). One could argue that scooping trail mix 20 or 40 times is not as strategic or efficient as scooping a few times and then reasoning proportionally to find a solution.

TABLE 1

The domain names change between grades 5 and 6, but the conceptual understanding builds from kindergarten through grade 8.

K–8 Domain Progressions in the CCSSM

Domains	K	1	2	3	4	5	6	7	8
Counting and Cardinality	■								
Operations and Algebraic Thinking	■	■	■	■	■	■			
Number and Operations in Base Ten	■	■	■	■	■	■			
Number and Operations-Fractions				■	■	■			
Ratio and Proportional Relationships							■	■	
The Number System							■	■	■
Expressions and Equations							■	■	■
Functions									■
Measurement and Data	■	■	■	■	■	■			
Geometry	■	■	■	■	■	■	■	■	■
Statistics and Probability							■	■	■

In a succeeding class discussion, however, students used repeated reasoning when they found how many bags could be filled with 1 cup of trail mix and then used multiplication to express the repeated addition (or repeated filling) of bags with the same serving size (SMP 8). The discussion included precise communication about strategies and math concepts (SMP 6). Steven constructed an argument (SMP 3) using quantitative reasoning (SMP 2). Students drew representations to model the situations with mathematics. Others wrote equations to match the situations, such as $5 \div 1/3 = 15$ (SMP 4).

Examining the domains in grades 5 and 6

The CCSSM grade-level Standards are organized in domains across each grade level. Although the domain names change between grades 5 and 6, the conceptual understanding within them builds one concept on another from kindergarten through grade 8 (see **table 1**). Doing mathematical work in the Operations and Algebraic Thinking Domain (K–grade 5) offers students foundational experiences and understanding for evaluating and solving equations in the Expressions and Equation Domain (grades 6–8). When we imagine students working from rich tasks, such as those featured in this article, it is not difficult to envision them creating an equation to express relationships within the problems or graphing the relationship between scoop size and number of bags as a means of generalizing their pattern. Likewise, student work within the Number and Operations in Base Ten Domain (K–grade 5) supports student development in the Number System Domain (grades 6–8). Learning to think flexibly about situations involving numbers and having ways of representing quantitative relationships can help students move into other number concepts, including integers. Furthermore, student fraction work (K–grade 5) supports the progression to working in the Ratio and Proportional Relationships Domain (grades 6–7). As both of our examples show, operations with fractions offer important opportunities to build the groundwork for proportional reasoning. In the Tupelo Township task, students could reason about the sizes of some sections of land relative to other sections. By using proportional reasoning in the Scooping Trail Mix task, students were able to reason about the impact of the scoop size on the number of bags that could be created.

Concluding thoughts

Cognitively demanding tasks (Smith and Stein 1998) provide an essential component for implementation of CCSSM in grades 5 and 6. These tasks engage students to better understand exploring concepts, processes, and relationships. Tasks of this sort allow students to meet both the Content Standards and the Standards of Mathematical Practice in engaging and challenging ways. As our examples show, cognitively demanding tasks engage students and encourage exploration of mathematics rather than repeated problem solving apart from a context. Use CCSSM to deepen your students' level of understanding of mathematical concepts while furthering their ability to engage in the practices of mathematicians.

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